APPENDIX ATTORNEY DOCKET NO.: 2925-0497P

Frequency Hopping with LCP Sequences

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May 12, 2001

Abstract

This paper introduces an approach for generating frequency hopping sequences to achieve a better frequency diversity performance

1 Introduction

The majority of the new features in GERAN is designed to support efficient packet switched service, which is characterized by bursty traffic. Compared to the circuit swiched voice traffic, the requirement for physical channel design is more demanding in terms of stable channel quality. As channel stablity for bursty traffic requires high frequency diversity in short time period. The present paper introduces a deterministic approach to generate frequency hopping sequence. New sequences show statistical property outperforming the GSM FH.

As measure for the frequency diversity we use a metrics called "repetition distance", which is defined as the minimum number of hops between two occurrences of the same frequency along a sequence. It is readily seen that the cyclic hopping ($HSN = 0$) provides the maximum repetition distance $n-1$ when there are n frequencies in the pool ($MA = n$). The repetition distance becomes 0 for non-frequency hopping. However, the cyclic frequency hopping as specified in GSM has adverse impact on the system performance, including on the interference diverstity. The goal of this paper is to introduce a sequence generator that can generate non-cyclic sequences with the maximum repetition distance. The algorithm is called "layered cyclic permutation" (LCP).

2 Algorithm

LCP sequences are generated in vector

$$a_0^{(k)}, a_2^{(k)}, ..., a_{n-1}^{(k)} \tag{1}$$

for discrete time $k = 1, 2, 3, ...,$, in two steps:

1. Generate a finite sequence for $k = 0, 1, ..., n-1$, where n is the number of available frequencies
2. Generate an infinite sequence using the finite sequence, i.e. $\{a_l^{(k)}\}_{l=0,k=0}^{n-1,n-1}$, generated by the first step.

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type 1						
$l = 0, 1, 2, .., 5$						(k)
3	2	5	4	1	0	(1)
4	5	0	1	2	3	(2)
1	0	3	2	5	4	(3)
2	3	4	5	0	1	(4)
5	4	1	0	3	2	(5)
0	1	2	3	4	5	(6)

type 2						
$l = 0, 1, 2, 3, 5$						(k)
1	0	3	2	5	4	(1)
2	3	4	5	0	1	(2)
5	4	1	0	3	2	(3)
0	1	2	3	4	5	(4)
3	2	5	4	1	0	(5)
4	5	0	1	2	3	(6)

Table 1: Blocks for $n = 6$ with initial value $\mathrm{x} = \{0, 1, 2, 3, 4, 5\}$: l refers to channels (row) and (k) refers to time (column).

2.1 Step 1: Basic Sequence

Assume $n = pq$, say $q = 2$ and $p = n/2$. Let $a_l^{(k)}$ indicate the index of frequency to hop at time k for channel l. Futhermore, l can be expressed by (i, j) such that $l = i \cdot q + j$. Starting with $a_l^{(0)} = a_l$ for $l = 0, 1, 2, .., n-1$, the value of $a_l^{(k)}$ at time $k > 0$ is determined by

$$a_l^{(k)} = a_{l_k} \tag{2}$$

where l_k is a function of k and l, and is determined through (i, j) by

$$l_k = [(i + k_1) \bmod p] \cdot q + (j + k_2) \bmod q \tag{3}$$

with $i = 0, 1, ..., p-1$ and $j = 0, 1, ..., q-1$, where[1]

$$\begin{aligned} k_1 &= [(k \bmod 2)(k+1)/2 + (1 - k \bmod 2)k/2] \bmod n & (4) \\ k_2 &= [(1 - k \bmod 2)k/2 + (k \bmod 2)(k-1)/2] \bmod n. & (5) \end{aligned}$$

Table 1 shows an example for $n = 6$. Since $n = 2 \cdot 3$ is a product of two mutual prime numbers as well as an even number, both type 1 and type 2 apply.

2.2 Step 2: Sequence of Longer Period

Assume $\{a_l^{(k)}\}_{l=0,k=0}^{n,n}$ is generated in the first step. Trivially, an infinite sequence can be generated block-wise, by randomly selecting initial values

$$\mathrm{x} = \{a_0^{(0)}, a_1^{(0)}, ..., a_{n-1}^{(0)}\} \tag{6}$$

for each block of n by n matrix. Or more simply, by repeating the same block via

$$s_l^{(t)} = a_l^{(k)} \text{ for } k = t \bmod n \tag{7}$$

a repetition distance $n-1$ and period n can be obtained. Obviously, there are many ways to extend the period of the sequence. For instance with

$$s_l^{(k)} = a_l^{(k+i) \bmod n} \text{ for } i \cdot n \leq k < (i+1) \cdot n, \tag{8}$$

for $i = 0, 1, 2, .., n-1-r$ and $k = 0, 1, 2, ...$, where r is the given repetition distance, a time sequence $\{s_l(t)\}_{t=0}^{\infty}$ with repetition distance $r < n-1$ can be generated which has a period

[1]Alternatively, $k_1 = k_2 = k$, when p and q are mutual prime, which is refered to as type 1. The other case is refered to as type 2.

$(n-r) \cdot n^2$. Here we notice a trade-off between the period length and the repetition distance: they are reciprocal. The longest period can be provided by a pseudo random sequence, while the repetition distance approaches zero. Under constraint optimization, one can fix the repetition distance to $n-k$ for a given $k > 1$ and develop schemes to maximize the period, e.g by means of pseudo-random process to select initial value or blocks, or vice versa.

2.3 Property of Basic LCP Sequence

As mechanism underlying the basic LCP sequence is group operation rather than random selection, the LCP sequences can be better understood when viewed in blocks as follows

- Each block is a square matrix $\{s_{i,j}\}_{i=0,j=0}^{n-1,n-1}$ of $n \times n$, where column index i indicates the time and the row index j indicates the vector components, representing e.g. channels
- No two colums are equal, i.e. $s_{i,j} \neq s_{i',j}$ for $i \neq i'$ and $j = 0, 1, .., n-1$. No two rows are equal, i.e. $s_{i,j} \neq s_{i,j'}$ for $j \neq j'$ and $i = 0, 1, ..., n-1$. If the rows represent channels, then no channel remains fixed when time advances in column. If the columns represent the freqency hopping sequences, then no two sequences contain the same frequency at the same time.
- Within a block each frequency occurrs only once in a row, and each frequency occurs only once in a column.
- Each row, as well as column, becomes cyclic, when $n = pq$ with $p = 1$ or $q = 1$.
- Sequences generated using different initial vectors are different.

3 Feasibility and Statistics

Not every n allows for LCP sequence with maximum repetition distance. The typical number of frequencies available for frequency hopping in GSM is less than 64. Thus, it is important to find out how many $n \leq 64$ exist which allow the algorithm to achieve the maximum repetion distance $n-1$. It is proven by means of group theory that the maximum repetition distance can be achieved by LCP, when n is a product of two mutual prime numbers or at least an even number. Here, the case of n being prime is excluded, although it allows for sequence with maxium repetition distane. This is because prime n can only achieve this repetition distance by a cyclic sequence, which is nothing new and not interesting. By analyzing non-prime numbers $n \leq 64$ with resepct to the possible decomposition, it turns out there are only 2 numbers not feasible for the algorithm, they are $n = 9$ and $n = 25$. Therefore, the LCP algorithm with maximum repetition distance is feasible for all non-prime $n \leq 64$ but $n = 9$ (repetition distance 5) and $n = 25$ (repetition distance 9).

The large repetition distance is not achieved at the cost of the intereference diverstity. The LCP sequences generated by different initial vectors are independent, meaning their collision probability is no more than two statistically random sequences. The following is an example of deployment scenario:

Example 1 *Let $n = 6$ with $(q, p) = (2, 3)$. There are $n! = 60$ possible intial vectors to choose from. Assume the network deployment require a 9 reuse. Then, $9 \times 6 = 54$ different sequences are required, among which 9 are initial vectors. Thus, $n = 6$ is capable of supporting reuse 9 with independent hopping sequences by maximum repetition distance. Table 2 shows 4 block of basic sequences, corresponding to 4 different initial vectors.*

$\mathbf{x} = \{0,1,2,3,4,5\}$						
$l = 0,1,2,3,4,5$						(k)
3	2	5	4	1	0	(1)
4	5	0	1	2	3	(2)
1	0	3	2	5	4	(3)
2	3	4	5	0	1	(4)
5	4	1	0	3	2	(5)
0	1	2	3	4	5	(6)

$\mathbf{x} = \{1,2,3,4,5,0\}$						
$l = 0,1,2,3,4,5$						(k)
4	3	0	5	2	1	(1)
5	0	1	2	3	4	(2)
2	1	4	3	0	5	(3)
3	4	5	0	1	2	(4)
0	5	2	1	4	3	(5)
1	2	3	4	5	0	(6)

$\mathbf{x} = \{2,1,4,3,0,5\}$						
$l = 0,1,2,3,4,5$						(k)
3	4	5	0	1	2	(1)
0	5	2	1	4	3	(2)
1	2	3	4	5	0	(3)
4	3	0	5	2	1	(4)
5	0	1	2	3	4	(5)
2	1	4	3	0	5	(6)

$\mathbf{x} = \{0,3,2,5,4,1\}$						
$l = 0,1,2,3,4,5$						(k)
5	2	1	4	3	0	(1)
4	1	0	3	2	5	(2)
3	0	5	2	1	4	(3)
2	5	4	1	0	3	(4)
1	4	3	0	5	2	(5)
0	3	2	5	4	1	(6)

Table 2: Sequences by $n = 6$ with $(q, p) = (2, 3)$

To demonstrate the performance of LCP sequences, let

$$r(k, \mathbf{x}, s) := \min_{t \in \mathbf{N}_0} \{t \mid s_{k+t}(\mathbf{x}) = s_k(\mathbf{x})\} \tag{9}$$

measure the repetition distance of sequence $\{s_k\}_{k=0}^{\infty}$, where k denotes time in frame and $\mathbf{x}$ is the initial vector. In practice

$$r_{mean} = \frac{1}{n_t}\frac{1}{n_x}\sum_{k=0}^{n_t}\sum_{i=1}^{n_x} r(k, \mathbf{x}_i, s) \tag{10}$$

is adequate to estimate the repetition distance, with

$$E[r(k, \mathbf{x})|\mathbf{X}] = \lim_{n_t \to \infty} r_{mean}, \tag{11}$$

where E refers expectation and $\mathbf{X}$ denotes the set of selected initial vector,i.e. $\mathbf{X} = \{\mathbf{x}_1, \mathbf{x}_2, ..., \mathbf{x}_{n_x}\}$; n_x is the number of vectors contained in $\mathbf{X}$. In addition,

$$r_{dev}^2 = \frac{1}{n_t}\frac{1}{n_x}\sum_{k=0}^{n_t}\sum_{i=1}^{n_x}[r(k, \mathbf{x}_i, s) - r_{mean}]^2 \tag{12}$$

estimates the variance of the repetition distance for sufficiently large n_t and n_x.

To assess the probability of co-frequency collision between sequences generated by different initial values, we use the co-frequency collision ratio for reference sequence k

$$C_k := \frac{1}{n_x n_t}\sum_{k'=1}^{n_x}\sum_{t=1}^{n_t} \delta(s_{k',t}, s_{k,t}) \tag{13}$$

where $\{s_{k,t}\}_{t=0}^{n_t}$ is generated by initial values x_k, $k = 1, 2, .., n_x$, respectively, and

$$\delta(u, v) = \begin{cases} 1 & \text{when } u = v \\ 0 & \text{when } u \neq v \end{cases} \tag{14}$$

The finite number n_t is the sample size and n_x is the number of initial values used in the evaluation. By $n_t \to \infty$ the limit of C_k shall be the co-frequency probability, which is indpendent of k for LCP sequences.

The LCP algorithm of generating frequency hopping sequences is compared with GSM sequence generator via

1. LCP with period n against
2. GSM random sequence generator

Since LCP is deterministic, the repetition distance is known, $r_{mean,LCP} = n - 1$. Every GSM sequence (except $HSN = 0$) is generated by pseudo random numbers. Assuming the random number is uniformly distributed, the ideal performance is $r_{mean,GSM} = n/2$. Thus, for $n > 2$ there is always $r_{mean,LCP} \geq r_{mean,GSM}$, where the equal sign applies when n is prime. In addition, $r_{dev,LCP} = 0$ while $r_{dev,GSM} > 0$.

Again, assume the uniform distribution of the frequencies along the sequence generated by a GSM sequence generator, the ideal co-frequency probability for GSM is $C_{GSM} = 1/n^2$, given a set of co-frequencies with n elements. As for LCP, which is generated in vector, the collision event should be evaluated between a reference sequence, i.e. a column of a reference $t \times n$ matrix, and all n sequences generated simultaneously by another initial vector, i.e. n columns of a $t \times n$ matrix generated by a different initial vector. As all n columns of any $t \times n$ matrice generated by LCP are orthorgonal, the reference sequence must have collision with exactly one of the other n sequences. That is a probability $1/n$. On the other hand, any element of the reference column of the refrerence $t \times n$ matrix moves to the same frequency with a probability $1/n$, resulting in a co-frequency collision probability $C_{LCP} = 1/n^2$. Since this is independent of the choice of the reference sequence and of the choice of the reference initial vector, conclusion can be drawn that, in terms of co-frequency collision, the LCP has the same performance as an ideal GSM random sequence generator.

4 Conclusion

Aiming at reducing the bursty occurences of same frequencies during frequency hopping, a new approach (LCP) is developed. The approach allows for frequency hopping sequences being generated vector-wise that demonstrate superior statistic property relevant to frequency hopping. Comparison shows, the LCP algorithm outperforms the current GSM FH sequence generator.

References

[Stage2] 3GPP TSG GERAN *Stage 2 Description for GERAN*, 3GPP.

[05.02] GSM 05.02, ETSI